**Advanced time-series analysis (University of Lund, Economic History Department)**
30 Jan-3 February and 26-30 March 2012
**Lecture 7** Conditional heteroscedasticity models: ARCH and GARCH techniques and their applications.

### *7.a Motivation and the basic ARCH model*

Standard time series models are concerned about estimating the conditional expected value of a process: $y_t = E(y_t \mid \mathbf{X}_t) + u_t = \beta_0 + \sum_{i=1}^{p} \beta_i x_{it} + u_t$

Where traditionally it is assumed that $u_t \sim N(0, \sigma_u^2)$, or at least that the residuals are homoscedastic ($\sigma_u^2$ is constant and does not depend on **X** or t) and serially uncorrelated (meaning that all information that can be used for prediction is incorporated in the model). I will refer to this equation as "level equation".

If the homoscedasticity condition was violated, we either had the standard errors adjusted (heteroscedasticity robust standard errors) or we used a Weighted Least Squares. This was done in order to have more efficient estimates of the level equation.

But the variance of the residual (or residual volatility) can also be of importance to us: it can be a measure of unpredictability or risk. While it is the most popular in financial econometrics, it has also been used in macroeconometrics (Engle 1982)[1] and economic history (Földvári and Van Leeuwen 2011)[2]. Also it is a very often observed fact that there are more and less volatile periods, that is, volatility may be clustered (in short: volatility changes over time). This needs to be modeled.

The idea is the following:

The variance of the error term in the level equation can be written as:

$\sigma_{u_t}^2 = \mathrm{var}(u_t \mid u_{t-1}, u_{t-2}, \ldots) = E\left[ (u_t - E(u_t))^2 \mid u_{t-1}, u_{t-2}, \ldots \right]$

where we already assume that the variance may change over time (that is why the subscript t is added). So what we assume is that the variance of the residual can be explained by the past variances. Since $E(u_t) = 0$, the above equation can be rewritten purely in terms of the squared residuals:

$\sigma_{u_t}^2 = E\left[ u_t^{\,2} \mid u_{t-1}, u_{t-2}, \ldots \right]$ so basically: $\sigma_{u_t}^2 = \alpha_0 + \sum_{j=1}^{m} \alpha_j u_{t-j}^2$ which is an Autoregressive Conditional Heteroscedasticity (ARCH(m)) model. If $\alpha_j > 0$ you have a clustering and also it is quite obvious that it will lead to a more leptokurtic (positive excess kurtosis) residuals from the level equation. Also, since residual variance is strictly positive, you have an assumption that the estimated (conditional) volatility is also positive.

The conditional volatility model is actually an estimation of the two equations:

the level equation: $y_t = \beta_0 + \sum_{i=1}^{p} \beta_i x_{it} + u_t$ with $u_t \sim N(0, \sigma_{u_t}^2)$

[1] Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50, 4: 987–1007.

[2] Földvári, P. and Van Leeuwen, B. (2011) Conditional heteroscedasticity in historical commodity price series Cliometrica, 5(2):165-186).

and the variance equation: $\sigma_{u_t}^2 = \alpha_0 + \sum_{j=1}^{m} \alpha_j u_{t-j}^2$

Very often the residual variance $\sigma_{u_t}^2$ is denoted by h.

Often the residual term from the level equation is written as:

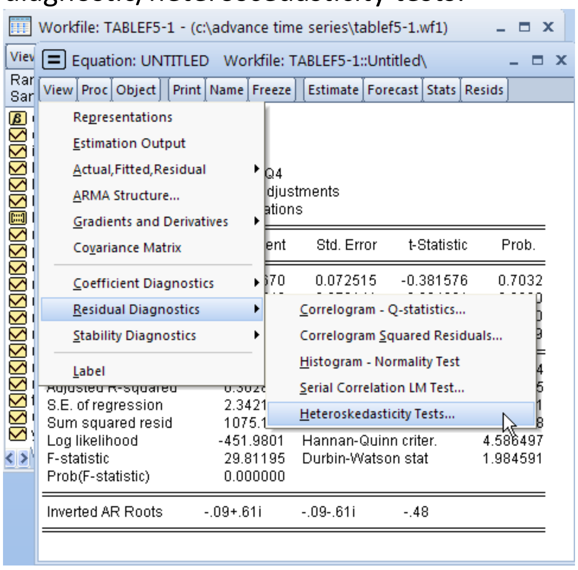
$u_t = \varepsilon_t \sigma_t = \varepsilon_t \sqrt{h_t} \ \ \varepsilon_t \sim N(0,1)$

where $\sigma_{u_t}^2 = h_t \ = \alpha_0 + \sum_{j=1}^{m} \alpha_j u_{t-j}^2$

Of course, you will use the squared residuals to obtain some knowledge regarding the volatility of the series that are not the same. So: $\sigma_{u_t}^2 = u_t^2 + e_t \ e_t \sim WN(0, \sigma_e^2)$. Basically you say here that your squared residual can be modeled by an AR(m) model: $u_t^2 = \alpha_0 + \sum_{j=1}^{m} \alpha_j u_{t-j}^2 + e_t$, where $e_t = u_t^2 - \sigma_{u_t}^2 \sim WN(0, \sigma_e^2)$ Of course, if your series is stationary, its variance should also be finite, so the variance equation should have the coefficient accordingly (it should be a stable process). If, for example, $\sigma_{u_t}^2 = \alpha_0 + u_{t-1}^2$, you will end up with a non-finite variance.

### 7.b Testing for ARCH type heteroscedasticity

The fundamental test for the need of a conditional heteroscedasticity model is one of the heteroscedasticity models and you can find it accordingly among the residual diagnostic/heteroscedasticity tests:




This requires however that you first have a model for the level equation.

If you have no structural model in mind you can always return to the Box-Jenkins methodology.

Fris tof all, make the best possible test using the Box-Jenkins method. For this example I use the Table f5.1 and look for an ARIMA type model for the inflation.

Since unit-root tests suggest that the inflation is I(1) I take its first-difference and go on with the specification. Finally it turns out that an AR(2) model fits the change of inflation fine so we choose a ARIMA(2,1,0) model for it:

Dependent Variable: D(INFL)
Method: Least Squares
Date: 03/21/12 Time: 19:52
Sample (adjusted): 1950Q4 2000Q4
Included observations: 201 after adjustments
Convergence achieved after 3 iterations

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 1.217108 | 0.345072 | 3.527111 | 0.0005 |
| @QUARTER=2 | -0.910511 | 0.566583 | -1.607023 | 0.1097 |
| @QUARTER=3 | -1.652897 | 0.503548 | -3.282504 | 0.0012 |
| @QUARTER=4 | -2.391285 | 0.563485 | -4.243744 | 0.0000 |
| AR(1) | -0.568147 | 0.066450 | -8.549951 | 0.0000 |
| AR(2) | -0.298428 | 0.066058 | -4.517689 | 0.0000 |

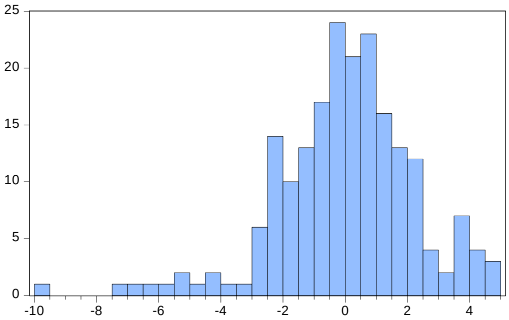
| | | | |
|---|---|---|---|
| R-squared | 0.354596 | Mean dependent var | -0.046490 |
| Adjusted R-squared | 0.338048 | S.D. dependent var | 2.798501 |
| S.E. of regression | 2.276873 | Akaike info criterion | 4.512880 |
| Sum squared resid | 1010.910 | Schwarz criterion | 4.611486 |
| Log likelihood | -447.5444 | Hannan-Quinn criter. | 4.552780 |
| F-statistic | 21.42731 | Durbin-Watson stat | 2.095259 |
| Prob(F-statistic) | 0.000000 | | |

| | | |
|---|---|---|
| Inverted AR Roots | -.28+.47i | -.28-.47i |

Date: 03/21/12 Time: 19:53
Sample: 1950Q4 2000Q4
Included observations: 201
Q-statistic probabilities adjusted for 2 ARMA term(s)

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | -0.062 | -0.062 | 0.7904 | |
| | | 2 | -0.069 | -0.073 | 1.7548 | |
| | | 3 | 0.020 | 0.011 | 1.8411 | 0.175 |
| | | 4 | 0.115 | 0.113 | 4.5639 | 0.102 |
| | | 5 | -0.084 | -0.068 | 6.0211 | 0.111 |
| | | 6 | 0.015 | 0.020 | 6.0653 | 0.194 |
| | | 7 | -0.038 | -0.051 | 6.3655 | 0.272 |
| | | 8 | -0.106 | -0.122 | 8.7304 | 0.189 |
| | | 9 | -0.083 | -0.090 | 10.187 | 0.178 |
| | | 10 | -0.017 | -0.054 | 10.252 | 0.248 |

Note that I found some seasonality in the series so I choose the simplest approach to correcting for this and included dummies for the quarters. The residual autocorrelation is not significant at 10% at any lags, so we can live with this.




Series: Residuals
Sample 1950Q4 2000Q4
Observations 201

| | |
|---|---|
| Mean | -5.24e-12 |
| Median | 0.210338 |
| Maximum | 4.997198 |
| Minimum | -9.995525 |
| Std. Dev. | 2.248232 |
| Skewness | -0.754793 |
| Kurtosis | 5.110088 |
| Jarque-Bera | 56.37484 |
| Probability | 0.000000 |

The residual is not normally distributed though, that may be because of some unmodelled structure of the volatility. The ARCH test is carried out with 4 lags:

Heteroskedasticity Test: ARCH

| | | | |
|---|---|---|---|
| F-statistic | 3.824008 | Prob. F(4,192) | 0.0051 |
| Obs*R-squared | 14.53630 | Prob. Chi-Square(4) | 0.0058 |

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/21/12 Time: 19:54
Sample (adjusted): 1951Q4 2000Q4
Included observations: 197 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 2.900535 | 0.684111 | 4.239861 | 0.0000 |
| RESID^2(-1) | 0.032477 | 0.071735 | 0.452732 | 0.6513 |
| RESID^2(-2) | 0.020561 | 0.052785 | 0.389533 | 0.6973 |
| RESID^2(-3) | 0.157193 | 0.052449 | 2.997091 | 0.0031 |
| RESID^2(-4) | 0.082233 | 0.053538 | 1.535982 | 0.1262 |

| | | | |
|---|---|---|---|
| R-squared | 0.073788 | Mean dependent var | 4.333514 |
| Adjusted R-squared | 0.054492 | S.D. dependent var | 7.542081 |
| S.E. of regression | 7.333710 | Akaike info criterion | 6.847893 |
| Sum squared resid | 10326.39 | Schwarz criterion | 6.931223 |
| Log likelihood | -669.5175 | Hannan-Quinn criter. | 6.881626 |
| F-statistic | 3.824008 | Durbin-Watson stat | 2.010262 |
| Prob(F-statistic) | 0.005150 | | |

The level equation is:

$$\Delta \inf_t = \beta_0 + \beta_1 \Delta \inf_{t-1} + \beta_2 \Delta \inf_{t-2} + \beta_4 D_t^{Q2} + \beta_5 D_t^{Q3} + \beta_6 D_t^{Q4} + \hat{u}_t$$

The test equation for the ARCH test is:

$$\hat{u}_t^2 = \hat{\alpha}_0 + \hat{\alpha}_1 \hat{u}_{t-1}^2 + \hat{\alpha}_2 \hat{u}_{t-2}^2 + \hat{\alpha}_3 \hat{u}_{t-3}^2 + \hat{\alpha}_4 \hat{u}_{t-4}^2 + e_t$$

Where in case of no ARCH type structure in the residual volatility, you should expect that $\hat{\alpha}_1 = \hat{\alpha}_2 = \hat{\alpha}_3 = \hat{\alpha}_4 = 0$ which is the null hypothesis of the test. So basically you carry out a joint significance test on the test equation.

In the above case we find that the null hypothesis is rejected at 1%, so some further analysis may be warranted.

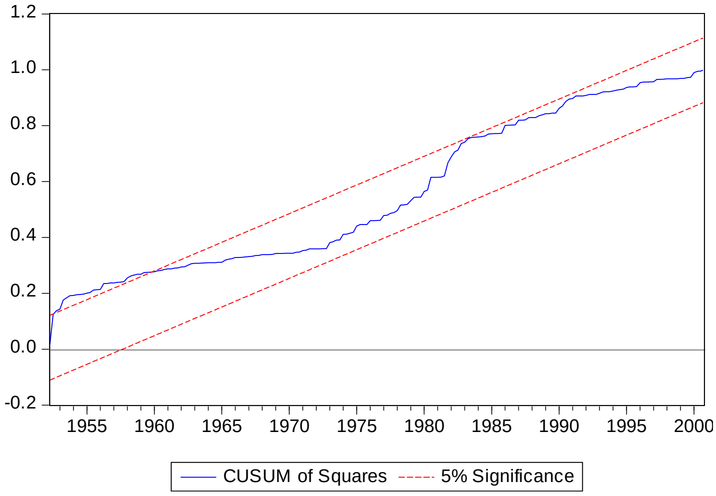
Alternatively, you can use the Q-test of the squared residuals:

Date: 03/26/12 Time: 10:42
Sample: 1950Q4 2000Q4
Included observations: 201

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.162 | 0.162 | 5.3603 | 0.021 |
| | | 2 | 0.157 | 0.134 | 10.410 | 0.005 |
| | | 3 | 0.182 | 0.145 | 17.256 | 0.001 |
| | | 4 | 0.116 | 0.057 | 20.067 | 0.000 |
| | | 5 | 0.116 | 0.056 | 22.864 | 0.000 |
| | | 6 | 0.035 | -0.032 | 23.124 | 0.001 |
| | | 7 | 0.076 | 0.033 | 24.353 | 0.001 |
| | | 8 | 0.009 | -0.037 | 24.372 | 0.002 |
| | | 9 | 0.070 | 0.054 | 25.421 | 0.003 |
| | | 10 | -0.020 | -0.056 | 25.508 | 0.004 |
| | | 11 | 0.024 | 0.020 | 25.630 | 0.007 |
| | | 12 | 0.008 | -0.012 | 25.644 | 0.012 |
| | | 13 | -0.007 | -0.003 | 25.653 | 0.019 |
| | | 14 | -0.009 | -0.021 | 25.672 | 0.028 |
| | | 15 | -0.034 | -0.023 | 25.920 | 0.039 |

This shows us that the squared residuals from the regression show a degree of autocorrelation.

Finally you can rely on CUSUMSQ test as well, but this time you check for a structural instability of the squared recursive residuals, so it can show you if residual variance depends on time:



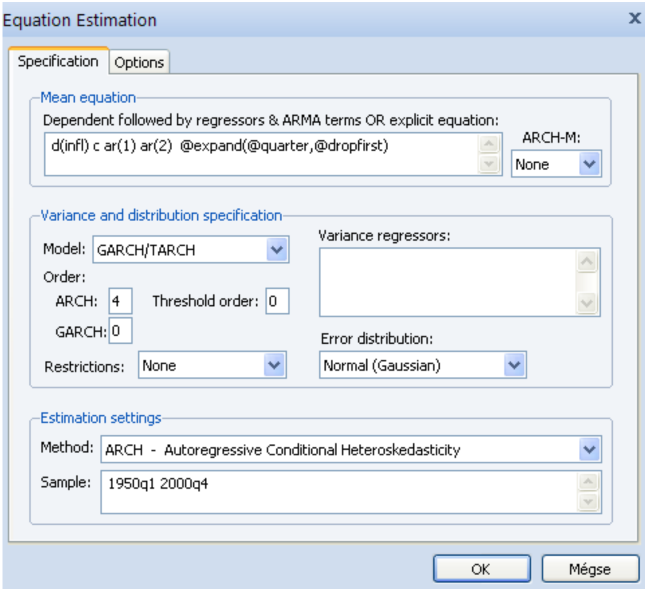

Again this suggests residual variance being dependent on time.

### 7.c. Estimating an ARCH model

There are two options to estimate an ARCH model. The first is to estimate the level and variance equations as a system (that is simultaneously). This can be done with a Maximum Likelihood method. Another option is to do the estimation in two steps. First you estimate the level equation with an OLS/WLS or ML (in case of an ARMA type model ML is preferable).

If you use a least-squares approach, it may be worthwhile to use a WLS because of the heteroscedasticity of the residual.
After this, you can use a standard regression technique again on the squared residuals from the level equation, which is basically the same as the ARCH test. When will you go for a two-step approach? Especially in historical analysis you often have a lot of missing observations in your sample. In standard softwares, like Eviews, ML algorithms are not capable of treating missing observations, so you will be forced to go for an alternative.
If you use the built-in ARCH estimation procedure of the Eviews you should specify both equations:

Now we tell the EViews that the level equation should be a ARIMA(3,1,0) type model, while the variance equation should be a ARCH(4) type. We also say that we believe that the errors follow a normal distribution. We obtain the following results:




Seemingly the ARCH(4) was not the best idea as we have two insignificant coefficients.

Dependent Variable: D(INFL)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/21/12 Time: 19:57
Sample (adjusted): 1950Q4 2000Q4
Included observations: 201 after adjustments
Convergence achieved after 64 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(7) + C(8)*RESID(-1)^2 + C(9)*RESID(-2)^2 + C(10)*RESID(-3)^2 + C(11)*RESID(-4)^2

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 1.132906 | 0.294009 | 3.853300 | 0.0001 |
| @QUARTER=2 | -0.724855 | 0.526412 | -1.376973 | 0.1685 |
| @QUARTER=3 | -1.541350 | 0.385132 | -4.002136 | 0.0001 |
| @QUARTER=4 | -2.361262 | 0.619934 | -3.808892 | 0.0001 |
| AR(1) | -0.645461 | 0.072588 | -8.892127 | 0.0000 |
| AR(2) | -0.220693 | 0.076888 | -2.870315 | 0.0041 |
| | | Variance Equation | | |
| C | 2.372884 | 0.430868 | 5.507218 | 0.0000 |
| RESID(-1)^2 | 0.104468 | 0.081250 | 1.285768 | 0.1985 |
| RESID(-2)^2 | 0.203012 | 0.086632 | 2.343387 | 0.0191 |
| RESID(-3)^2 | -0.012946 | 0.053154 | -0.243558 | 0.8076 |
| RESID(-4)^2 | 0.201220 | 0.112670 | 1.785929 | 0.0741 |

| | | | |
|---|---|---|---|
| R-squared | 0.338391 | Mean dependent var | -0.046490 |
| Adjusted R-squared | 0.321427 | S.D. dependent var | 2.798501 |
| S.E. of regression | 2.305281 | Akaike info criterion | 4.401536 |
| Sum squared resid | 1036.292 | Schwarz criterion | 4.582313 |
| Log likelihood | -431.3543 | Hannan-Quinn criter. | 4.474686 |
| Durbin-Watson stat | 1.963664 | | |

| | | |
|---|---|---|
| Inverted AR Roots | -.32+.34i | -.32-.34i |

We can start to improve the model, and look at the information criteria for a lead in selection.

You can for example assume that there is seasonality in the variance as well, meaning that in some quarters you tend to have higher or lower variance of the residual.

Dependent Variable: D(INFL)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/21/12 Time: 20:07
Sample (adjusted): 1950Q4 2000Q4
Included observations: 201 after adjustments
Convergence achieved after 38 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(7) + C(8)*RESID(-1)^2 + C(9)*RESID(-2)^2 + C(10)*RESID(-3)^2 + C(11)*RESID(-4)^2

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.990775 | 0.389353 | 2.544670 | 0.0109 |
| @QUARTER=2 | -0.512658 | 0.691535 | -0.741334 | 0.4585 |
| @QUARTER=3 | -1.443367 | 0.365718 | -3.946663 | 0.0001 |
| @QUARTER=4 | -2.107415 | 0.591905 | -3.560395 | 0.0004 |
| AR(1) | -0.655560 | 0.062571 | -10.47700 | 0.0000 |
| AR(2) | -0.142266 | 0.067871 | -2.096108 | 0.0361 |
| | | Variance Equation | | |
| C | 5.675703 | 1.761057 | 3.222895 | 0.0013 |
| RESID(-1)^2 | 0.059828 | 0.087319 | 0.685170 | 0.4932 |
| RESID(-2)^2 | 0.349835 | 0.102478 | 3.413749 | 0.0006 |
| RESID(-3)^2 | -0.000705 | 0.063967 | -0.011014 | 0.9912 |
| RESID(-4)^2 | 0.057668 | 0.085258 | 0.676396 | 0.4988 |
| @QUARTER=2 | -3.439228 | 1.815070 | -1.894819 | 0.0581 |
| @QUARTER=3 | -4.502706 | 1.812494 | -2.484260 | 0.0130 |
| @QUARTER=4 | -4.356478 | 1.834437 | -2.374831 | 0.0176 |

| | | | |
|---|---|---|---|
| R-squared | 0.313311 | Mean dependent var | -0.046490 |
| Adjusted R-squared | 0.295704 | S.D. dependent var | 2.798501 |
| S.E. of regression | 2.348568 | Akaike info criterion | 4.356437 |
| Sum squared resid | 1075.576 | Schwarz criterion | 4.586518 |
| Log likelihood | -423.8219 | Hannan-Quinn criter. | 4.449538 |
| Durbin-Watson stat | 1.982553 | | |

| | | |
|---|---|---|
| Inverted AR Roots | -.33-.19i | -.33+.19i |

We find that in the 2nd, 3rd and 4th quarters the volatility is lower than in the first. This is not very surprising, since most price changes occur in the beginning of the year.

You can also add or remove lags. In this case we find that an ARCH(5) model has the lowest AIC and SBC statistics.

Dependent Variable: D(INFL)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/21/12 Time: 21:17
Sample (adjusted): 1950Q4 2000Q4
Included observations: 201 after adjustments
Failure to improve Likelihood after 44 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(7) + C(8)*RESID(-1)^2 + C(9)*RESID(-2)^2 + C(10)*RESID(-3)^2 + C(11)*RESID(-4)^2 + C(12)*RESID(-5)^2

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.772291 | 0.375487 | 2.056775 | 0.0397 |
| @QUARTER=2 | -0.440661 | 0.680544 | -0.647514 | 0.5173 |
| @QUARTER=3 | -1.183635 | 0.381422 | -3.103218 | 0.0019 |
| @QUARTER=4 | -1.732877 | 0.513043 | -3.377647 | 0.0007 |
| AR(1) | -0.669078 | 0.036979 | -18.09362 | 0.0000 |
| AR(2) | -0.171599 | 0.047686 | -3.598536 | 0.0003 |
| | | Variance Equation | | |
| C | 5.875675 | 1.779126 | 3.302562 | 0.0010 |
| RESID(-1)^2 | -0.009866 | 0.056182 | -0.175610 | 0.8606 |
| RESID(-2)^2 | 0.230926 | 0.110896 | 2.082378 | 0.0373 |
| RESID(-3)^2 | -0.026323 | 0.049885 | -0.527661 | 0.5977 |
| RESID(-4)^2 | 0.096752 | 0.091672 | 1.055415 | 0.2912 |
| RESID(-5)^2 | 0.168250 | 0.084971 | 1.980084 | 0.0477 |
| @QUARTER=2 | -3.977172 | 1.793031 | -2.218128 | 0.0265 |
| @QUARTER=3 | -4.570041 | 1.858094 | -2.459532 | 0.0139 |
| @QUARTER=4 | -5.127838 | 1.772449 | -2.893080 | 0.0038 |

| | | | |
|---|---|---|---|
| R-squared | 0.314309 | Mean dependent var | -0.046490 |
| Adjusted R-squared | 0.296727 | S.D. dependent var | 2.798501 |
| S.E. of regression | 2.346861 | Akaike info criterion | 4.289331 |
| Sum squared resid | 1074.013 | Schwarz criterion | 4.535846 |
| Log likelihood | -416.0777 | Hannan-Quinn criter. | 4.389081 |
| Durbin-Watson stat | 1.951206 | | |
| Inverted AR Roots | -.33-.24i | -.33+.24i | |

The residual has no serial correlation significant at 1%:

Date: 03/21/12 Time: 21:17
Sample: 1950Q4 2000Q4
Included observations: 201
Q-statistic probabilities adjusted for 2 ARMA term(s)

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | -0.022 | -0.022 | 0.0960 | |
| | | 2 | -0.102 | -0.103 | 2.2424 | |
| | | 3 | 0.072 | 0.069 | 3.3244 | 0.068 |
| | | 4 | 0.126 | 0.121 | 6.6294 | 0.036 |
| | | 5 | -0.085 | -0.068 | 8.1464 | 0.043 |
| | | 6 | 0.023 | 0.040 | 8.2586 | 0.083 |
| | | 7 | -0.034 | -0.066 | 8.5024 | 0.131 |
| | | 8 | -0.106 | -0.111 | 10.884 | 0.092 |
| | | 9 | -0.113 | -0.116 | 13.584 | 0.059 |
| | | 10 | -0.045 | -0.082 | 14.016 | 0.081 |
| | | 11 | -0.024 | -0.019 | 14.141 | 0.117 |
| | | 12 | 0.046 | 0.073 | 14.607 | 0.147 |

Some are significant at 5% though. You can add a further lag of the dependent variable to the level equation.

Dependent Variable: D(INFL)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/21/12 Time: 21:20
Sample (adjusted): 1951Q1 2000Q4
Included observations: 200 after adjustments
Failure to improve Likelihood after 20 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(8) + C(9)*RESID(-1)^2 + C(10)*RESID(-2)^2 + C(11)*RESID(-3)^2 + C(12)*RESID(-4)^2 + C(13)*RESID(-5)^2

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.927012 | 0.450562 | 2.057458 | 0.0396 |
| @QUARTER=2 | -0.693129 | 0.868333 | -0.798229 | 0.4247 |
| @QUARTER=3 | -1.336473 | 0.471617 | -2.833812 | 0.0046 |
| @QUARTER=4 | -1.975558 | 0.630679 | -3.132429 | 0.0017 |
| AR(1) | -0.739872 | 0.064948 | -11.39170 | 0.0000 |
| AR(2) | -0.365289 | 0.085480 | -4.273412 | 0.0000 |
| AR(3) | -0.213277 | 0.068433 | -3.116566 | 0.0018 |
| Variance Equation | | | | |
| C | 5.584444 | 1.481224 | 3.770154 | 0.0002 |
| RESID(-1)^2 | -0.014776 | 0.034152 | -0.432650 | 0.6653 |
| RESID(-2)^2 | 0.199839 | 0.100202 | 1.994368 | 0.0461 |
| RESID(-3)^2 | -0.018096 | 0.032020 | -0.565150 | 0.5720 |
| RESID(-4)^2 | 0.135498 | 0.097645 | 1.387657 | 0.1652 |
| RESID(-5)^2 | 0.142758 | 0.068628 | 2.080160 | 0.0375 |
| @QUARTER=2 | -3.564372 | 1.536017 | -2.320528 | 0.0203 |
| @QUARTER=3 | -4.325761 | 1.594279 | -2.713302 | 0.0067 |
| @QUARTER=4 | -5.097462 | 1.511509 | -3.372433 | 0.0007 |

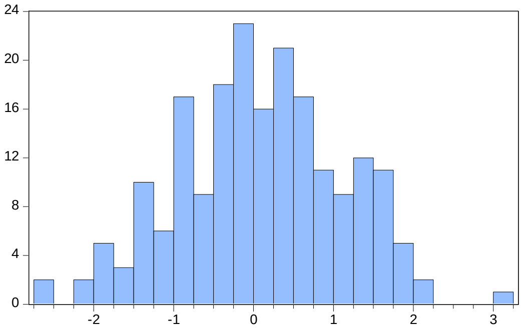
| | | | |
|---|---|---|---|
| R-squared | 0.346455 | Mean dependent var | -0.042844 |
| Adjusted R-squared | 0.326138 | S.D. dependent var | 2.805045 |
| S.E. of regression | 2.302637 | Akaike info criterion | 4.221734 |
| Sum squared resid | 1023.312 | Schwarz criterion | 4.485600 |
| Log likelihood | -406.1734 | Hannan-Quinn criter. | 4.328517 |
| Durbin-Watson stat | 1.853424 | | |

| | | | |
|---|---|---|---|
| Inverted AR Roots | -.04-.56i | -.04+.56i | -.67 |

Date: 03/21/12 Time: 21:21
Sample: 1951Q1 2000Q4
Included observations: 200
Q-statistic probabilities adjusted for 3 ARMA term(s)

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.017 | 0.017 | 0.0592 | |
| | | 2 | 0.036 | 0.036 | 0.3262 | |
| | | 3 | 0.176 | 0.175 | 6.6481 | |
| | | 4 | 0.039 | 0.034 | 6.9569 | 0.008 |
| | | 5 | -0.028 | -0.042 | 7.1144 | 0.029 |
| | | 6 | 0.016 | -0.018 | 7.1649 | 0.067 |
| | | 7 | -0.077 | -0.090 | 8.4033 | 0.078 |
| | | 8 | -0.140 | -0.135 | 12.552 | 0.028 |
| | | 9 | -0.123 | -0.122 | 15.747 | 0.015 |
| | | 10 | -0.097 | -0.068 | 17.749 | 0.013 |
| | | 11 | -0.068 | -0.009 | 18.729 | 0.016 |
| | | 12 | 0.009 | 0.068 | 18.745 | 0.027 |

But this will not help you much.




Series: Standardized Residuals
Sample 1951Q1 2000Q4
Observations 200

| | |
|---|---|
| Mean | 0.079501 |
| Median | 0.090113 |
| Maximum | 3.100286 |
| Minimum | -2.608892 |
| Std. Dev. | 1.012874 |
| Skewness | -0.083539 |
| Kurtosis | 2.829529 |
| Jarque-Bera | 0.474793 |
| Probability | 0.788679 |

The residuals seem normally distributed though.

We can also check if there is any ARCH type heteroscedasticity remaining in the residuals:

Heteroskedasticity Test: ARCH

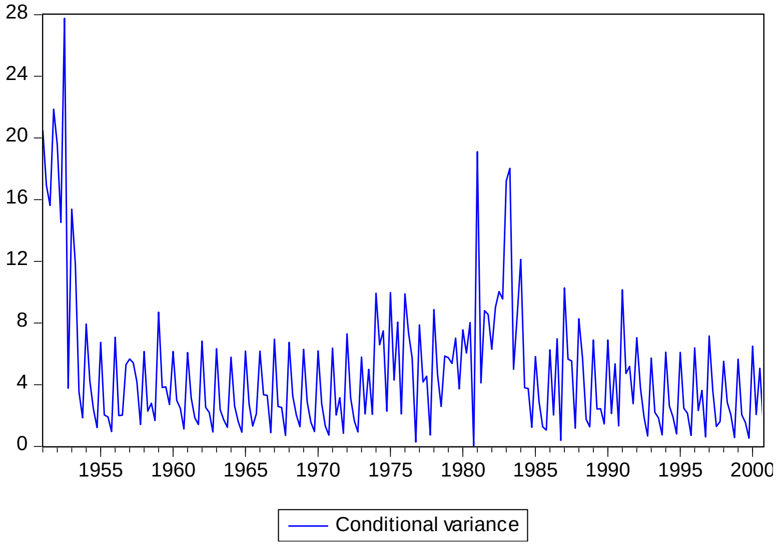
| | | | |
|---|---|---|---|
| F-statistic | 0.694496 | Prob. F(4,191) | 0.5966 |
| Obs*R-squared | 2.809837 | Prob. Chi-Square(4) | 0.5901 |

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 03/21/12 Time: 21:22
Sample (adjusted): 1952Q1 2000Q4
Included observations: 196 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.910731 | 0.169985 | 5.357713 | 0.0000 |
| WGT_RESID^2(-1) | 0.061617 | 0.072361 | 0.851529 | 0.3955 |
| WGT_RESID^2(-2) | -0.041867 | 0.072151 | -0.580267 | 0.5624 |
| WGT_RESID^2(-3) | 0.099590 | 0.071469 | 1.393472 | 0.1651 |
| WGT_RESID^2(-4) | -0.021861 | 0.071795 | -0.304487 | 0.7611 |

| | | | |
|---|---|---|---|
| R-squared | 0.014336 | Mean dependent var | 1.009918 |
| Adjusted R-squared | -0.006306 | S.D. dependent var | 1.379002 |
| S.E. of regression | 1.383344 | Akaike info criterion | 3.512064 |
| Sum squared resid | 365.5053 | Schwarz criterion | 3.595689 |
| Log likelihood | -339.1822 | Hannan-Quinn criter. | 3.545919 |
| F-statistic | 0.694496 | Durbin-Watson stat | 1.973177 |
| Prob(F-statistic) | 0.596613 | | |

Our estimate for the residual variance can also be graphed:




Showing that the volatility of inflation was especially large during the first half of the 1980s and the beginning of our sample, the early 1950s.

**7.d GARCH: motivation and estimation**

Bollerslev introduced a more general formation of the conditional heteroscedasticity models. The GARCH (p,m) model looks as follows:

$$\sigma_{u_t}^2 = h_t = \alpha_0 + \sum_{i=1}^{p} \gamma_i \sigma_{u_{t-i}}^2 + \sum_{j=1}^{m} \alpha_j u_{t-j}^2$$

since $e_t = u_t^2 - \sigma_{u_t}^2$,we can rewrite this as:

$$u_t^2 = \alpha_0 + \sum_{i=1}^{p} \gamma_i \sigma_{u_{t-i}}^2 + \sum_{j=1}^{m} \alpha_j u_{t-j}^2 + e_t = \alpha_0 + \sum_{i=1}^{p} \gamma_i u_{t.i}^2 + \sum_{j=1}^{m} \alpha_j u_{t-j}^2 + e_t - \sum_{i=1}^{p} \gamma_i e_{t-i}$$

which is an ARMA(max(p,m),p) representation of the square residual.

In case of a GARCH(1,1) we assume that:

$\sigma_{u_t}^2 = \alpha_0 + \gamma_1 \sigma_{u_{t-1}}^2 + \alpha_1 u_{t-1}^2$, which is equivalent with believing that

$u_t^2 = \alpha_0 + (\alpha_1 + \gamma_1) u_{t-1}^2 + e_t - \gamma_1 e_{t-1}$ or

$\sigma_{u_t}^2 = \alpha_0 + (\gamma_1 + \alpha_1) \sigma_{u_{t-1}}^2 + \alpha_1 e_{t-1}$

Obviously, this model is stationary if $\alpha_1 + \gamma_1 < 1$. If this is not the case, the residual variance (or the volatility) is not finite.

A GARCH model is usually estimated by a ML procedure.

The above model can be re-estimated as a GARCH model as follows:

Dependent Variable: D(INFL)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/22/12 Time: 14:48
Sample (adjusted): 1951Q1 2000Q4
Included observations: 200 after adjustments
Convergence achieved after 40 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(8) + C(9)*RESID(-1)^2 + C(10)*GARCH(-1)

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.901551 | 0.450929 | 1.999320 | 0.0456 |
| @QUARTER=2 | -0.495605 | 0.849858 | -0.583161 | 0.5598 |
| @QUARTER=3 | -1.380489 | 0.443816 | -3.110496 | 0.0019 |
| @QUARTER=4 | -1.717142 | 0.697461 | -2.461989 | 0.0138 |
| AR(1) | -0.726351 | 0.081088 | -8.957599 | 0.0000 |
| AR(2) | -0.334132 | 0.095639 | -3.493673 | 0.0005 |
| AR(3) | -0.199581 | 0.081265 | -2.455929 | 0.0141 |
| | | Variance Equation | | |
| C | 4.564941 | 1.577484 | 2.893811 | 0.0038 |
| RESID(-1)^2 | 0.115425 | 0.048571 | 2.376422 | 0.0175 |
| GARCH(-1) | 0.775049 | 0.074079 | 10.46241 | 0.0000 |
| @QUARTER=2 | -7.262906 | 2.784994 | -2.607871 | 0.0091 |
| @QUARTER=3 | -4.257227 | 1.747629 | -2.436002 | 0.0149 |
| @QUARTER=4 | -5.123921 | 1.823769 | -2.809522 | 0.0050 |

| | | | |
|---|---|---|---|
| R-squared | 0.344733 | Mean dependent var | -0.042844 |
| Adjusted R-squared | 0.324362 | S.D. dependent var | 2.805045 |
| S.E. of regression | 2.305669 | Akaike info criterion | 4.256644 |
| Sum squared resid | 1026.009 | Schwarz criterion | 4.471035 |
| Log likelihood | -412.6644 | Hannan-Quinn criter. | 4.343405 |
| Durbin-Watson stat | 1.893810 | | |

| | | | |
|---|---|---|---|
| Inverted AR Roots | -.03+.54i | -.03-.54i | -.67 |

The Akaike Information Criterion suggest that the ARCH(5) fits the data better than a GARCH(1,1) but the Schwarz criterion suggest that the GARCH is the better model. A GARCH(2,1) will perform even better:

Dependent Variable: D(INFL)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/22/12 Time: 14:50
Sample (adjusted): 1951Q1 2000Q4
Included observations: 200 after adjustments
Convergence achieved after 114 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(8) + C(9)*RESID(-1)^2 + C(10)*GARCH(-1) + C(11)*GARCH(-2)

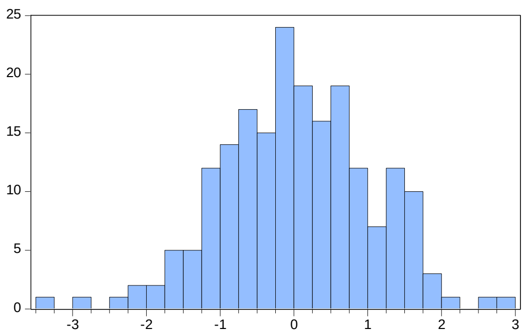
| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.921125 | 0.415876 | 2.214901 | 0.0268 |
| @QUARTER=2 | -0.417032 | 0.741255 | -0.562603 | 0.5737 |
| @QUARTER=3 | -1.389687 | 0.479989 | -2.895245 | 0.0038 |
| @QUARTER=4 | -2.002786 | 0.639554 | -3.131537 | 0.0017 |
| AR(1) | -0.708214 | 0.072621 | -9.752238 | 0.0000 |
| AR(2) | -0.359943 | 0.081921 | -4.393797 | 0.0000 |
| AR(3) | -0.167350 | 0.077555 | -2.157806 | 0.0309 |
| | | Variance Equation | | |
| C | 5.615685 | 1.386307 | 4.050822 | 0.0001 |
| RESID(-1)^2 | 0.036350 | 0.017507 | 2.076264 | 0.0379 |
| GARCH(-1) | 1.549517 | 0.098583 | 15.71782 | 0.0000 |
| GARCH(-2) | -0.655416 | 0.072552 | -9.033748 | 0.0000 |
| @QUARTER=2 | -11.95909 | 3.451115 | -3.465283 | 0.0005 |
| @QUARTER=3 | -2.145363 | 0.571290 | -3.755293 | 0.0002 |
| @QUARTER=4 | -7.281530 | 2.377345 | -3.062884 | 0.0022 |

| | | | |
|---|---|---|---|
| R-squared | 0.361257 | Mean dependent var | -0.042844 |
| Adjusted R-squared | 0.341400 | S.D. dependent var | 2.805045 |
| S.E. of regression | 2.276412 | Akaike info criterion | 4.246066 |
| Sum squared resid | 1000.136 | Schwarz criterion | 4.476948 |
| Log likelihood | -410.6066 | Hannan-Quinn criter. | 4.339500 |
| Durbin-Watson stat | 1.895977 | | |

| | | | |
|---|---|---|---|
| Inverted AR Roots | -.06-.53i | -.06+.53i | -.58 |

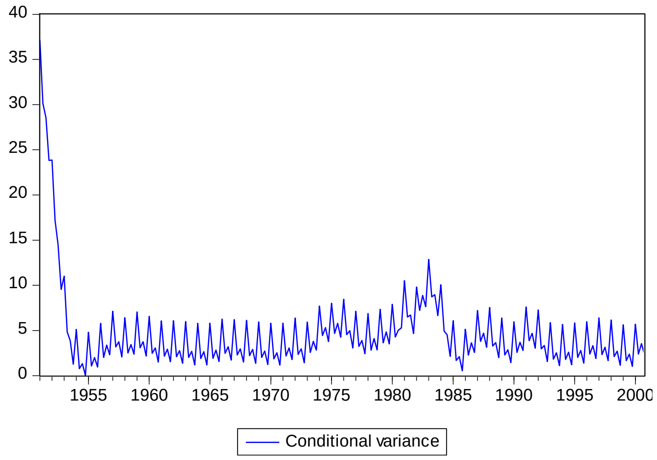
Date: 03/22/12 Time: 14:53
Sample: 1951Q1 2000Q4
Included observations: 200
Q-statistic probabilities adjusted for 3 ARMA term(s)

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.019 | 0.019 | 0.0709 | |
| | | 2 | 0.036 | 0.035 | 0.3319 | |
| | | 3 | 0.109 | 0.108 | 2.7580 | |
| | | 4 | 0.065 | 0.061 | 3.6206 | 0.057 |
| | | 5 | -0.014 | -0.024 | 3.6641 | 0.160 |
| | | 6 | -0.015 | -0.031 | 3.7114 | 0.294 |
| | | 7 | -0.109 | -0.123 | 6.1991 | 0.185 |
| | | 8 | -0.122 | -0.123 | 9.3169 | 0.097 |
| | | 9 | -0.108 | -0.098 | 11.780 | 0.067 |
| | | 10 | -0.087 | -0.057 | 13.402 | 0.063 |
| | | 11 | -0.088 | -0.044 | 15.049 | 0.058 |
| | | 12 | 0.047 | 0.091 | 15.520 | 0.078 |




Series: Standardized Residuals
Sample 1951Q1 2000Q4
Observations 200

| | |
|---|---|
| Mean | 0.031804 |
| Median | 0.017190 |
| Maximum | 2.851424 |
| Minimum | -3.277575 |
| Std. Dev. | 1.008829 |
| Skewness | -0.107196 |
| Kurtosis | 3.226851 |
| Jarque-Bera | 0.811880 |
| Probability | 0.666350 |



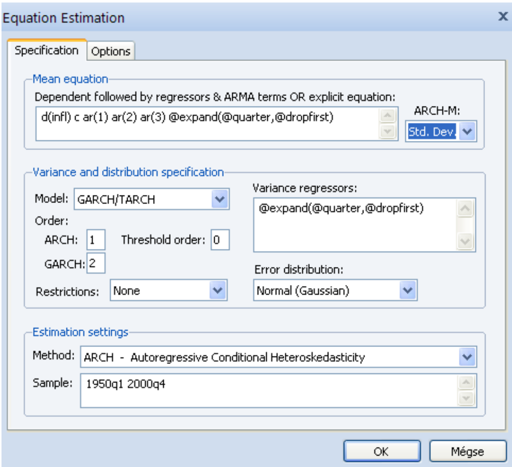

## 7.e. Further extensions of the methodology

There are a lot of possible extensions of the conditional heteroscedasticity methodology.

*Interrelation between variance and level (GARCH-M)*

We can assume that higher volatility has an effect on the level. This means a small modification of the level equation:

$$y_t = \beta_0 + \sum_{i=1}^{p} \beta_i x_{it} + \lambda \sigma_{u_t} + u_t$$




Dependent Variable: D(INFL)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/22/12 Time: 15:03
Sample (adjusted): 1951Q1 2000Q4
Included observations: 200 after adjustments
Convergence achieved after 46 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(9) + C(10)*RESID(-1)^2 + C(11)*GARCH(-1) + C(12)*GARCH(-2)

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| @SQRT(GARCH) | -0.514075 | 0.238460 | -2.155808 | 0.0311 |
| C | 2.618890 | 1.034634 | 2.531224 | 0.0114 |
| @QUARTER=2 | -1.431268 | 1.086876 | -1.316864 | 0.1879 |
| @QUARTER=3 | -1.864605 | 0.564667 | -3.302132 | 0.0010 |
| @QUARTER=4 | -2.703709 | 0.967540 | -2.794416 | 0.0052 |
| AR(1) | -0.753192 | 0.070975 | -10.61213 | 0.0000 |
| AR(2) | -0.359499 | 0.082642 | -4.350101 | 0.0000 |
| AR(3) | -0.198358 | 0.076475 | -2.593758 | 0.0095 |
| Variance Equation | | | | |
| C | 5.556949 | 1.701908 | 3.265129 | 0.0011 |
| RESID(-1)^2 | 0.051289 | 0.034439 | 1.489281 | 0.1364 |
| GARCH(-1) | 1.460726 | 0.224192 | 6.515517 | 0.0000 |
| GARCH(-2) | -0.584372 | 0.172579 | -3.386115 | 0.0007 |
| @QUARTER=2 | -11.08967 | 4.074413 | -2.721784 | 0.0065 |
| @QUARTER=3 | -3.162071 | 1.156543 | -2.734073 | 0.0063 |
| @QUARTER=4 | -6.892989 | 2.724430 | -2.530067 | 0.0114 |

| | | | |
|---|---|---|---|
| R-squared | 0.381702 | Mean dependent var | -0.042844 |
| Adjusted R-squared | 0.359160 | S.D. dependent var | 2.805045 |
| S.E. of regression | 2.245509 | Akaike info criterion | 4.225352 |
| Sum squared resid | 968.1236 | Schwarz criterion | 4.472726 |
| Log likelihood | -407.5352 | Hannan-Quinn criter. | 4.325460 |
| Durbin-Watson stat | 1.944235 | | |

| | | | |
|---|---|---|---|
| Inverted AR Roots | -.05-.55i | -.05+.55i | -.66 |

We find here that in periods with high volatility inflation tends to be lower.

*Threshold ARCH and GARCH (T-ARCH/T-GARCH)*

It is also possible that the previous period's volatility has asymmetrical effect on today's volatility depending on the movement of the dependent variable. In other words, if we had a positive shock in inflation ($u_{t-1}$ is positive) than it is going to have a bigger impact on volatility now than a negative shock. This requires that we adjust the variance equation a bit:

$$\sigma_{u_t}^2 = \alpha_0 + \gamma_1 \sigma_{u_{t-1}}^2 + \alpha_1 u_{t-1}^2 + \alpha_2 \left( I(u_{t-1} < 1) \cdot u_{t-1}^2 \right)$$

Where $I(u_{t-1} < 1)$ is an indicator variable (a dummy) taking the value one only if the previous period's residual was negative.

Dependent Variable: D(INFL)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/22/12 Time: 15:36
Sample (adjusted): 1951Q1 2000Q4
Included observations: 200 after adjustments
Convergence achieved after 64 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(8) + C(9)*RESID(-1)^2 + C(10)*RESID(-1)^2*(RESID(-1)<0) + C(11)*GARCH(-1) + C(12)*GARCH(-2)

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 1.121377 | 0.455734 | 2.460595 | 0.0139 |
| @QUARTER=2 | -0.798675 | 0.814223 | -0.980905 | 0.3266 |
| @QUARTER=3 | -1.552376 | 0.434964 | -3.568975 | 0.0004 |
| @QUARTER=4 | -2.024757 | 0.679984 | -2.977657 | 0.0029 |
| AR(1) | -0.760761 | 0.065695 | -11.58027 | 0.0000 |
| AR(2) | -0.398581 | 0.080791 | -4.933476 | 0.0000 |
| AR(3) | -0.203313 | 0.075691 | -2.686103 | 0.0072 |
| | | Variance Equation | | |
| C | 5.233534 | 1.494539 | 3.501773 | 0.0005 |
| RESID(-1)^2 | 0.115808 | 0.050772 | 2.280963 | 0.0226 |
| RESID(-1)^2*(RESID(-1)<0) | -0.094008 | 0.053062 | -1.771658 | 0.0765 |
| GARCH(-1) | 1.483559 | 0.103049 | 14.39665 | 0.0000 |
| GARCH(-2) | -0.595250 | 0.082853 | -7.184391 | 0.0000 |
| @QUARTER=2 | -11.46700 | 3.418010 | -3.354877 | 0.0008 |
| @QUARTER=3 | -1.893425 | 1.042680 | -1.815922 | 0.0694 |
| @QUARTER=4 | -6.769010 | 2.286765 | -2.960081 | 0.0031 |

| | | | |
|---|---|---|---|
| R-squared | 0.350893 | Mean dependent var | -0.042844 |
| Adjusted R-squared | 0.330714 | S.D. dependent var | 2.805045 |
| S.E. of regression | 2.294805 | Akaike info criterion | 4.232063 |
| Sum squared resid | 1016.363 | Schwarz criterion | 4.479437 |
| Log likelihood | -408.2063 | Hannan-Quinn criter. | 4.332172 |
| Durbin-Watson stat | 1.797498 | | |

| | | | |
|---|---|---|---|
| Inverted AR Roots | -.06-.56i | -.06+.56i | -.64 |

The results suggest that the effect of a shock on the volatility of inflation does depend on the direction of the shock. Negative shock to inflation has a much lower effect on volatility, than positive shocks. Actually, with a Wald-test we can even find out that a negative shock has no impact on volatility at all while a positive shock does.

Wald Test:
Equation: Untitled

| Test Statistic | Value | df | Probability |
|---|---|---|---|
| t-statistic | 1.385036 | 185 | 0.1677 |
| F-statistic | 1.918325 | (1, 185) | 0.1677 |
| Chi-square | 1.918325 | 1 | 0.1660 |

Null Hypothesis: C(9) + C(10)=0
Null Hypothesis Summary:

| Normalized Restriction (= 0) | Value | Std. Err. |
|---|---|---|
| C(9) + C(10) | 0.021800 | 0.015740 |

Restrictions are linear in coefficients.

*Exponential GARCH (EGARCH)*

The variance equation is modified as follows (EGARCH(1,1):

$\ln \sigma_{u_t}^2 = \alpha_0 + \gamma_1 \ln \sigma_{u_{t-1}}^2 + \alpha_1 \left| \frac{u_{t-1}}{\sigma_{u_{t-1}}} \right|$ or if you expect an asymmetric effect of shocks:

$$\ln \sigma_{u_t}^2 = \alpha_0 + \gamma_1 \ln \sigma_{u_{t-1}}^2 + \alpha_1 \left| \frac{u_{t-1}}{\sigma_{u_{t-1}}} \right| + \lambda_1 \alpha_1 \frac{u_{t-1}}{\sigma_{u_{t-1}}}$$

where, if $\lambda_1 \neq 0$, there is an asymmetry. Additional advantage is the you have can only have positive estimated values of conditional volatility.

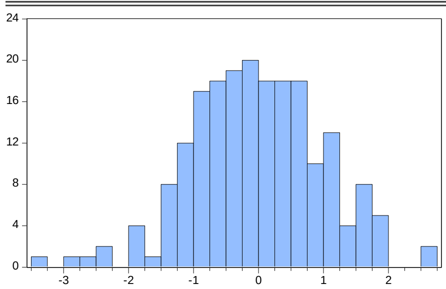
Dependent Variable: D(INFL)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/26/12 Time: 11:56
Sample (adjusted): 1951Q1 2000Q4
Included observations: 200 after adjustments
Convergence achieved after 44 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(8) + C(9)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(10)
*RESID(-1)/@SQRT(GARCH(-1)) + C(11)*LOG(GARCH(-1))

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 1.630831 | 0.486514 | 3.352076 | 0.0008 |
| @QUARTER=2 | -1.653426 | 0.900873 | -1.835358 | 0.0665 |
| @QUARTER=3 | -1.900318 | 0.449841 | -4.224419 | 0.0000 |
| @QUARTER=4 | -2.767523 | 0.737193 | -3.754138 | 0.0002 |
| AR(1) | -0.737258 | 0.077318 | -9.535454 | 0.0000 |
| AR(2) | -0.376033 | 0.081053 | -4.639325 | 0.0000 |
| AR(3) | -0.249981 | 0.076194 | -3.280838 | 0.0010 |
| | Variance Equation | | | |
| C(8) | 0.954632 | 0.250110 | 3.816849 | 0.0001 |
| C(9) | 0.313581 | 0.120152 | 2.609876 | 0.0091 |
| C(10) | 0.175131 | 0.081945 | 2.137180 | 0.0326 |
| C(11) | 0.944324 | 0.037607 | 25.11023 | 0.0000 |
| C(12) | -1.822741 | 0.454974 | -4.006254 | 0.0001 |
| C(13) | -1.186893 | 0.396441 | -2.993872 | 0.0028 |
| C(14) | -1.555944 | 0.453715 | -3.429339 | 0.0006 |

| | | | |
|---|---|---|---|
| R-squared | 0.338050 | Mean dependent var | -0.042844 |
| Adjusted R-squared | 0.317471 | S.D. dependent var | 2.805045 |
| S.E. of regression | 2.317397 | Akaike info criterion | 4.239608 |
| Sum squared resid | 1036.474 | Schwarz criterion | 4.470490 |
| Log likelihood | -409.9608 | Hannan-Quinn criter. | 4.333042 |
| Durbin-Watson stat | 1.862751 | | |
| Inverted AR Roots | -.02+.59i | -.02-.59i | -.71 |

Date: 03/26/12 Time: 11:57
Sample: 1951Q1 2000Q4
Included observations: 200
Q-statistic probabilities adjusted for 3 ARMA term(s)

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | -0.002 | -0.002 | 0.0010 | |
| | | 2 | 0.009 | 0.009 | 0.0183 | |
| | | 3 | 0.157 | 0.157 | 5.0946 | |
| | | 4 | 0.017 | 0.018 | 5.1548 | 0.023 |
| | | 5 | -0.005 | -0.008 | 5.1600 | 0.076 |
| | | 6 | -0.009 | -0.035 | 5.1785 | 0.159 |
| | | 7 | -0.114 | -0.123 | 7.9163 | 0.095 |
| | | 8 | -0.099 | -0.103 | 9.9911 | 0.075 |
| | | 9 | -0.110 | -0.109 | 12.562 | 0.051 |
| | | 10 | -0.117 | -0.088 | 15.488 | 0.030 |
| | | 11 | -0.123 | -0.097 | 18.715 | 0.016 |
| | | 12 | 0.036 | 0.071 | 18.988 | 0.025 |
| | | 13 | -0.016 | 0.023 | 19.045 | 0.040 |
| | | 14 | -0.043 | -0.017 | 19.440 | 0.054 |
| | | 15 | -0.052 | -0.094 | 20.019 | 0.067 |
| | | 16 | 0.106 | 0.066 | 22.483 | 0.048 |
| | | 17 | 0.043 | 0.011 | 22.898 | 0.062 |
| | | 18 | 0.045 | 0.019 | 23.350 | 0.077 |
| | | 19 | -0.108 | -0.176 | 25.953 | 0.055 |
| | | 20 | 0.106 | 0.069 | 28.487 | 0.040 |
| | | 21 | 0.063 | 0.042 | 29.392 | 0.044 |




Series: Standardized Residuals
Sample 1951Q1 2000Q4
Observations 200

| | |
|---|---|
| Mean | -0.032158 |
| Median | -0.113136 |
| Maximum | 2.628276 |
| Minimum | -3.274285 |
| Std. Dev. | 0.998506 |
| Skewness | -0.100080 |
| Kurtosis | 3.301729 |
| Jarque-Bera | 1.092539 |
| Probability | 0.579106 |

## 7.f. Some applications

*Forecasting risk:*

We take the Dow Jones index at close between Jan 1990 and Apr 2011 (dataset is dj.wf1). This is a monthly data, normally we would use high frequency data (daily or weekly) but this will do now. The log difference of the index is a kind of weighted monthly returns on the stocks traded at NYSE. The returns are stationary:

Null Hypothesis: R is stationary
Exogenous: Constant, Linear Trend
Bandwidth: 3 (Newey-West automatic) using Bartlett kernel

| | | LM-Stat. |
|---|---|---|
| Kwiatkowski-Phillips-Schmidt-Shin test statistic | | 0.061448 |
| Asymptotic critical values*: | 1% level | 0.216000 |
| | 5% level | 0.146000 |
| | 10% level | 0.119000 |

*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

| | |
|---|---|
| Residual variance (no correction) | 0.001845 |
| HAC corrected variance (Bartlett kernel) | 0.001858 |

And has no serial correlations:

Date: 03/22/12 Time: 16:38
Sample: 1990M01 2011M04
Included observations: 255

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | 0.026 | 0.026 | 0.1711 | 0.679 |
| 2 | -0.025 | -0.026 | 0.3388 | 0.844 |
| 3 | 0.024 | 0.025 | 0.4850 | 0.922 |
| 4 | 0.052 | 0.051 | 1.2033 | 0.878 |
| 5 | 0.020 | 0.018 | 1.3033 | 0.935 |
| 6 | -0.099 | -0.099 | 3.9019 | 0.690 |
| 7 | 0.085 | 0.090 | 5.8321 | 0.559 |
| 8 | 0.054 | 0.042 | 6.6105 | 0.579 |
| 9 | -0.026 | -0.023 | 6.7963 | 0.658 |
| 10 | -0.054 | -0.047 | 7.5858 | 0.669 |
| 11 | 0.037 | 0.034 | 7.9548 | 0.717 |
| 12 | 0.122 | 0.104 | 11.978 | 0.447 |
| 13 | -0.107 | -0.097 | 15.055 | 0.304 |
| 14 | -0.037 | -0.023 | 15.437 | 0.349 |
| 15 | -0.018 | -0.037 | 15.523 | 0.414 |
| 16 | 0.043 | 0.034 | 16.041 | 0.450 |
| 17 | -0.008 | 0.010 | 16.057 | 0.520 |

This seems to nicely confirm the weak form of the efficient market hypothesis.
The distribution of the returns is not normal though:

Empirical Distribution Test for R
Hypothesis: Normal
Date: 03/22/12 Time: 16:39
Sample (adjusted): 1990M02 2011M04
Included observations: 255 after adjustments

| Method | Value | Adj. Value | Probability |
|---|---|---|---|
| Lilliefors (D) | 0.074351 | NA | 0.0016 |
| Cramer-von Mises (W2) | 0.318493 | 0.319118 | 0.0002 |
| Watson (U2) | 0.257271 | 0.257775 | 0.0005 |
| Anderson-Darling (A2) | 1.889484 | 1.895107 | 0.0001 |

Method: Maximum Likelihood - d.f. corrected (Exact Solution)

| Parameter | Value | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| MU | 0.006122 | 0.002703 | 2.264783 | 0.0235 |
| SIGMA | 0.043166 | 0.001915 | 22.53886 | 0.0000 |

| | | | |
|---|---|---|---|
| Log likelihood | 440.0598 | Mean dependent var. | 0.006122 |
| No. of Coefficients | 2 | S.D. dependent var. | 0.043166 |

Heteroskedasticity Test: ARCH

| | | | |
|---|---|---|---|
| F-statistic | 3.551272 | Prob. F(4,246) | 0.0077 |
| Obs*R-squared | 13.70257 | Prob. Chi-Square(4) | 0.0083 |

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/22/12 Time: 16:40
Sample (adjusted): 1990M06 2011M04
Included observations: 251 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.001143 | 0.000298 | 3.838275 | 0.0002 |
| RESID^2(-1) | 0.107917 | 0.062913 | 1.715350 | 0.0875 |
| RESID^2(-2) | 0.031343 | 0.063025 | 0.497305 | 0.6194 |
| RESID^2(-3) | 0.093404 | 0.063026 | 1.481982 | 0.1396 |
| RESID^2(-4) | 0.148028 | 0.062921 | 2.352603 | 0.0194 |

| | | | |
|---|---|---|---|
| R-squared | 0.054592 | Mean dependent var | 0.001859 |
| Adjusted R-squared | 0.039219 | S.D. dependent var | 0.003529 |
| S.E. of regression | 0.003459 | Akaike info criterion | -8.475823 |
| Sum squared resid | 0.002944 | Schwarz criterion | -8.405595 |

And we find evidence for an ARCH type structure in the volatility of the returns.
An GARCH(1,1) model seems to fit the data nicely:

Dependent Variable: R
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/22/12 Time: 16:43
Sample (adjusted): 1990M02 2011M04
Included observations: 255 after adjustments
Convergence achieved after 17 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.007179 | 0.002576 | 2.786951 | 0.0053 |
| | | Variance Equation | | |
| C | 6.86E-05 | 4.64E-05 | 1.479173 | 0.1391 |
| RESID(-1)^2 | 0.132155 | 0.040863 | 3.234094 | 0.0012 |
| GARCH(-1) | 0.839703 | 0.040907 | 20.52709 | 0.0000 |

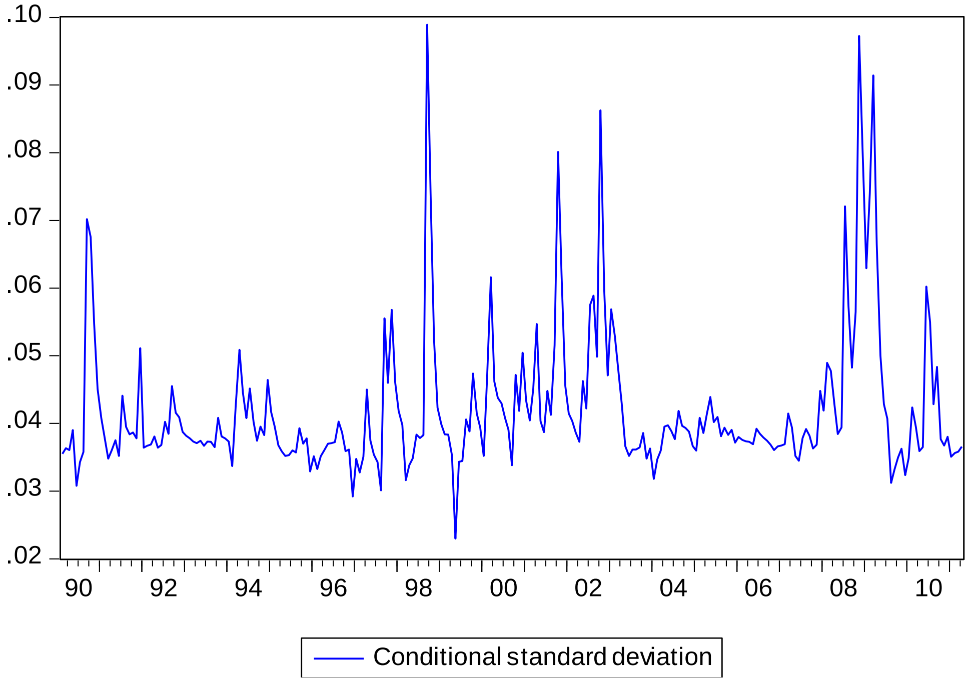
| | | | |
|---|---|---|---|
| R-squared | -0.000601 | Mean dependent var | 0.006122 |
| Adjusted R-squared | -0.000601 | S.D. dependent var | 0.043166 |
| S.E. of regression | 0.043179 | Akaike info criterion | -3.511108 |
| Sum squared resid | 0.473564 | Schwarz criterion | -3.455559 |
| Log likelihood | 451.6663 | Hannan-Quinn criter. | -3.488764 |
| Durbin-Watson stat | 1.947148 | | |

Dependent Variable: R
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/22/12 Time: 16:44
Sample (adjusted): 1990M02 2011M04
Included observations: 255 after adjustments
Convergence achieved after 19 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) +
C(5)*GARCH(-1)

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.006000 | 0.002747 | 2.184242 | 0.0289 |
| | | Variance Equation | | |
| C | 0.000697 | 0.000276 | 2.529705 | 0.0114 |
| RESID(-1)^2 | -0.094067 | 0.054311 | -1.732026 | 0.0833 |
| RESID(-1)^2*(RESID(-1)<0) | 0.383116 | 0.154018 | 2.487472 | 0.0129 |
| GARCH(-1) | 0.497961 | 0.201954 | 2.465712 | 0.0137 |

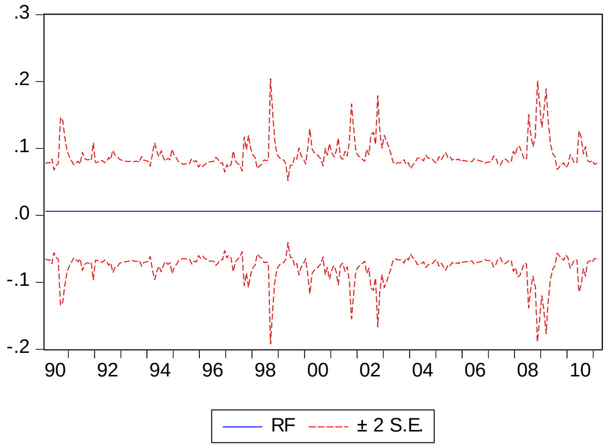
| | | | |
|---|---|---|---|
| R-squared | -0.000008 | Mean dependent var | 0.006122 |
| Adjusted R-squared | -0.000008 | S.D. dependent var | 0.043166 |
| S.E. of regression | 0.043166 | Akaike info criterion | -3.512065 |
| Sum squared resid | 0.473283 | Schwarz criterion | -3.442628 |
| Log likelihood | 452.7883 | Hannan-Quinn criter. | -3.484135 |
| Durbin-Watson stat | 1.948304 | | |

We find that a TGARCH model is even better, and negative shocks in returns seem to have had a bigger impact on volatility than positive shocks. This is referred to as leverage effect in the literature: price falls can have larger impact on volatility than price increases.

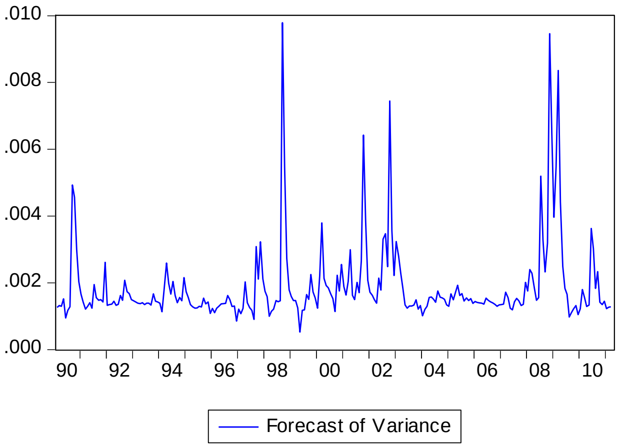



We can observe that certain periods were especially volatile, like the first half of 1998 or, not surprisingly, mid-2008.

One thing we can do with an ARCH/GARCH model is to forecast how risky the next period is going to be. We can use a static forecast to see how good the model would have performed in one period ahead forecasts:




| | |
|---|---|
| Forecast: RF | |
| Actual: R | |
| Forecast sample: 1990M01 2011M04 | |
| Adjusted sample: 1990M02 2011M04 | |
| Included observations: 255 | |
| Root Mean Squared Error | 0.043081 |
| Mean Absolute Error | 0.032276 |
| Mean Abs. Percent Error | 177.3381 |
| Theil Inequality Coefficient | 0.870079 |
| Bias Proportion | 0.000008 |
| Variance Proportion | NA |
| Covariance Proportion | NA |




*Looking for trends in market performance:*

In economic history the working of markets has become a focal point. Event hough this is often labeled as market efficiency, we should rather distinguish it from standard market efficiency (as suggested by Fama 1965)[3]. Normally economic historians look at the volatility of prices as a measure of the degree of integration and generally the efficiency of the markets in coping with risks.
As long as this exercises is done on cross-sectional data (that is, one looks at prices within the same period but from different places) we have nothing to add here. Still, it became increasingly popular to look not only on the spatial but also on the dynamic aspect of price volatility, where some special time-series problems may appear. For a detailed description you can consult Földvári and Van Leeuwen (2011).[4]

In case of cross sectional data, it is a standard exercise to calculate the Coefficient of Variation (CV) of the observed prices. The CV is the ratio of the standard deviation and the mean. If we have reason to believe that your observations come from the same probability distribution, that is, there exist a finite mean and standard deviation of them, this method

[3] Fama, E. F. (1965) The Behavior of Stock-Market Prices. The Journal of Business, Vol. 38, No. 1., 34-105.
[4] Földvári, P. and Van Leeuwen, B. (2011) Conditional heteroscedasticity in historical commodity price series Cliometrica, 5(2):165-186).

works fine. For example, you can calculate the CV for the price of grain for 50 cities in Europe in 1800 and in 1850 and then compare the two CVs.
Recently more authors started to look at the CV measured over time. So, they take single geographical unit, say a single city, and calculate the CV of the prices for a given period. Then they repeat this for another interval of time and they draw conclusions based on the observed differences. In the cited article we argue that this is incorrect. To see why, we need to accept that in most cases prices tend to exhibit trend-like behavior. This can be due to trend-stationarity but also if prices are random-walks with drift. The latter is more in line with the classical understanding of the efficient markets, but in case of historical prices you can find stationary prices as well.
Let us assume that the logarithm of a price ($p_t$) is a random-walk with drift process:

$$\ln p_t = \alpha + \ln p_{t-1} + \varepsilon_t$$

Using logarithm is not only due to some nice properties of taking logarithm, but because the standard deviation of the log prices is very close to the CV of the level of prices. So what we can find out about the standard deviation of the log prices is also true for the CV of the prices, without logartihmization.
We already know that the above process can be rewritten as:

$\ln p_t = \ln p_0 + \alpha t + \sum_{i=1}^{t} \varepsilon_i$ if $\varepsilon_t \sim WN(0,\sigma_\varepsilon^2)$ we would obtain the well-known relationship:

$\sigma_{\ln p_t}^2 = t\sigma_{\varepsilon_t}^2$ or $\sigma_{\ln p_t} = \sqrt{t}\sigma_{\varepsilon_t}$ where we already used the idea that $E(\ln p_t) = \ln p_0 + \alpha t$.
Obviously, both moments (expected value and variance) depends on time. That is, there is no unique, finite mean or variance. The method of Coefficient of Variation depends on the assumption that you have a variance and a mean that is constant over time. This is obviously not right.
So what should we expect? The longer the sample, the higher the variance gets. This has nothing to do with any “efficiency” of the markets whatsoever, or a detrimental process in integration of markets. This comes simply because the underlying prices are not mean-reverting and this inflates variance.
One solution is to difference prices before calculating their variance. This is a possible way to go, but it will only work if your price series are indeed classic random-walk processes. If not, because the DGP looks like this.

$\ln p_t = \alpha_0 + \alpha_1 \ln p_{t-1} + \alpha_2 \ln p_{t-2} + \varepsilon_t$, first-differencing will yield the following:

$\Delta \ln p_t = \alpha_0 + (\alpha_1 - 1)\ln p_{t-1} + \alpha_2 \ln p_{t-2} + \varepsilon_t$ so your estimated volatility of log p will have more than what you really wished for:

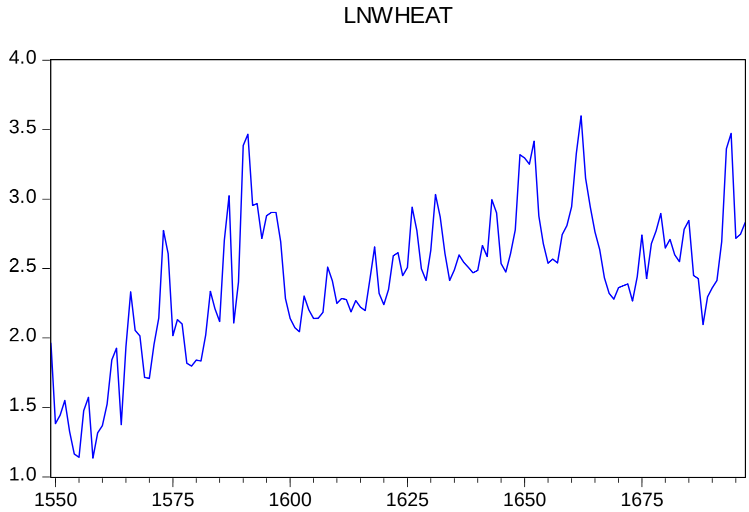
$$\sigma_{\Delta \ln p_t}^2 = (\alpha_1 - 1)^2 \sigma_{\ln p_{t-1}}^2 + \alpha_2^2 \sigma_{\ln p_{t-2}}^2 + \sigma_{\varepsilon_t}^2$$

For this reason the first step should be a modeling of the expected value of the prices (estimating a level equation) than you can further estimate the residual variance (variance equation). In other words, you should rather trust an ARCH/GARCH framework once you would like to learn something about the time trend of the volatility of prices. The volatility can be thought to be a combination of purely random shocks ($\theta$) that is assumed to be a white noise process and a multiplier that reflect the market’s ability to cope with this shocks ($\lambda_t$).

$\varepsilon_t = \theta_t \sqrt{\lambda_t}$, where $\theta_t \sim WN(0,\sigma_\theta^2)$

If we find lambda to decrease over time, we have evidence for a better performance of the market to cope with the unexpected.

To test this we have now the price of wheat in Paris 1549-1697.




There is an obvious upward trend in the data. If you used the standard methodology you would find something like this:

| sample | std deviation of log wheat (roughly the CV) | t |
|---|---|---|
| 1549-1697 | 0.503 | 149 |
| 1549-1580 | 0.406 | 32 |
| 1581-1640 | 0.333 | 60 |
| 1641-1697 | 0.400 | 57 |

The most striking finding is that the volatility of the whole sample, that we would expect to be some kind of average of the volatility of the sub-samples, is actually higher than any of the sub-samples. The reason is obvious: the primitive method will yield biased estimates. If one still believed in the above results, one would come to the conclusion that there was some improvement in terms of volatility during the first half of the $17^{th}$ century, but later this turned to the opposite. One way is to take the first differences:

Dependent Variable: D(LNWHEAT)
Method: Least Squares
Date: 03/24/12 Time: 10:18
Sample (adjusted): 1550 1697
Included observations: 148 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.005856 | 0.022222 | 0.263512 | 0.7925 |

| | | | |
|---|---|---|---|
| R-squared | 0.000000 | Mean dependent var | 0.005856 |
| Adjusted R-squared | 0.000000 | S.D. dependent var | 0.270338 |
| S.E. of regression | 0.270338 | Akaike info criterion | 0.228449 |
| Sum squared resid | 10.74318 | Schwarz criterion | 0.248700 |
| Log likelihood | -15.90523 | Hannan-Quinn criter. | 0.236677 |
| Durbin-Watson stat | 1.913972 | | |

The estimated standard deviation is now 0.270, much lower than with the above method. Still, even this can be misleading as this residual variance may contain a lot of information that could not really be brought into connection with any kind of market performance. So we find the best ARMA model. The big question is whether we should difference the log prices or not:

Null Hypothesis: LNWHEAT has a unit root
Exogenous: Constant, Linear Trend
Lag length: 2 (Spectral GLS-detrended AR based on SIC, maxlag=13)
Sample: 1549 1697
Included observations: 149

| | | MZa | MZt | MSB | MPT |
|---|---|---|---|---|---|
| Ng-Perron test statistics | | -20.2310 | -3.17892 | 0.15713 | 4.51400 |
| Asymptotic critical values*: | 1% | -23.8000 | -3.42000 | 0.14300 | 4.03000 |
| | 5% | -17.3000 | -2.91000 | 0.16800 | 5.48000 |
| | 10% | -14.2000 | -2.62000 | 0.18500 | 6.67000 |

*Ng-Perron (2001, Table 1)

| | |
|---|---|
| HAC corrected variance (Spectral GLS-detrended AR) | 0.042528 |

The best known unit-root test (Ng-Perron) cannot help us out here as it can reject the unit root hypothesis at 5% but not at 1% (MZa and MZt) and it can reject stationarity only at 1% (MSB and MPT).
Luckily, it does not make much difference if we just take the first difference of the prices independently whether it has a unit-root or not. Nevertheless, with historical data you should be careful not to difference unless it is needed since the ratio of measurement errors to signal may increase as a result of differencing. The seemingly best model has only a second-order autoregressive term:

Dependent Variable: D(LNWHEAT)
Method: Least Squares
Date: 03/24/12 Time: 10:25
Sample (adjusted): 1552 1697
Included observations: 146 after adjustments
Convergence achieved after 3 iterations

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.008310 | 0.014876 | 0.558614 | 0.5773 |
| AR(2) | -0.379175 | 0.075655 | -5.011920 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.148530 | Mean dependent var | 0.009510 |
| Adjusted R-squared | 0.142617 | S.D. dependent var | 0.267713 |
| S.E. of regression | 0.247889 | Akaike info criterion | 0.061929 |
| Sum squared resid | 8.848621 | Schwarz criterion | 0.102801 |
| Log likelihood | -2.520838 | Hannan-Quinn criter. | 0.078536 |
| F-statistic | 25.11935 | Durbin-Watson stat | 1.888415 |
| Prob(F-statistic) | 0.000002 | | |

But even this has already reduced the residual standard deviation to 0.248, so our fears that taking simply first-difference may not be enough (can be a good approximation, though) is justified. There is no apparent serial correlation in the residual:

Date: 03/24/12 Time: 10:28
Sample: 1552 1697
Included observations: 146
Q-statistic probabilities adjusted for 1 ARMA term(s)

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.052 | 0.052 | 0.4088 | |
| | | 2 | -0.048 | -0.051 | 0.7519 | 0.386 |
| | | 3 | -0.028 | -0.023 | 0.8731 | 0.646 |
| | | 4 | -0.116 | -0.116 | 2.9075 | 0.406 |
| | | 5 | -0.117 | -0.110 | 5.0157 | 0.286 |
| | | 6 | 0.003 | 0.001 | 5.0170 | 0.414 |
| | | 7 | -0.121 | -0.144 | 7.3107 | 0.293 |
| | | 8 | -0.024 | -0.034 | 7.4036 | 0.388 |
| | | 9 | -0.048 | -0.094 | 7.7718 | 0.456 |
| | | 10 | 0.045 | 0.025 | 8.0985 | 0.524 |
| | | 11 | -0.004 | -0.053 | 8.1012 | 0.619 |
| | | 12 | -0.053 | -0.096 | 8.5496 | 0.663 |
| | | 13 | 0.036 | 0.016 | 8.7561 | 0.724 |
| | | 14 | -0.016 | -0.064 | 8.7977 | 0.788 |
| | | 15 | -0.096 | -0.110 | 10.321 | 0.738 |
| | | 16 | -0.071 | -0.126 | 11.148 | 0.742 |
| | | 17 | 0.113 | 0.096 | 13.270 | 0.653 |
| | | 18 | 0.033 | -0.014 | 13.458 | 0.705 |

Date: 03/24/12 Time: 10:28
Sample: 1552 1697
Included observations: 146
Q-statistic probabilities adjusted for 1 ARMA term(s)

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.021 | 0.021 | 0.0659 | |
| | | 2 | 0.227 | 0.227 | 7.7933 | 0.005 |
| | | 3 | -0.112 | -0.127 | 9.6916 | 0.008 |
| | | 4 | 0.166 | 0.130 | 13.871 | 0.003 |
| | | 5 | 0.002 | 0.045 | 13.871 | 0.008 |
| | | 6 | -0.007 | -0.094 | 13.878 | 0.016 |
| | | 7 | -0.060 | -0.030 | 14.435 | 0.025 |
| | | 8 | 0.032 | 0.045 | 14.592 | 0.042 |
| | | 9 | 0.024 | 0.020 | 14.680 | 0.066 |
| | | 10 | -0.019 | -0.037 | 14.736 | 0.098 |
| | | 11 | 0.024 | 0.046 | 14.828 | 0.138 |
| | | 12 | -0.005 | 0.000 | 14.832 | 0.190 |
| | | 13 | 0.095 | 0.065 | 16.301 | 0.178 |
| | | 14 | -0.043 | -0.038 | 16.604 | 0.218 |
| | | 15 | 0.101 | 0.076 | 18.283 | 0.194 |
| | | 16 | -0.040 | -0.017 | 18.553 | 0.235 |
| | | 17 | 0.065 | -0.005 | 19.259 | 0.255 |
| | | 18 | -0.032 | 0.011 | 19.429 | 0.304 |

But the correlogram of the squared residuals (right) indicate some ARCH type behavior.
Now we can turn to a ARCH/GARCH model:

Dependent Variable: D(LNWHEAT)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/24/12 Time: 10:30
Sample (adjusted): 1552 1697
Included observations: 146 after adjustments
Convergence achieved after 18 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*@TREND

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.014347 | 0.016727 | 0.857686 | 0.3911 |
| AR(2) | -0.248794 | 0.089548 | -2.778344 | 0.0055 |
| | | Variance Equation | | |
| C | 0.053972 | 0.013599 | 3.968784 | 0.0001 |
| RESID(-1)^2 | -0.018400 | 0.069509 | -0.264711 | 0.7912 |
| RESID(-2)^2 | 0.200070 | 0.125419 | 1.595210 | 0.1107 |
| @TREND | -6.63E-05 | 0.000114 | -0.582283 | 0.5604 |

| | | | |
|---|---|---|---|
| R-squared | 0.130255 | Mean dependent var | 0.009510 |
| Adjusted R-squared | 0.124216 | S.D. dependent var | 0.267713 |
| S.E. of regression | 0.250535 | Akaike info criterion | 0.069326 |
| Sum squared resid | 9.038536 | Schwarz criterion | 0.191940 |
| Log likelihood | 0.939228 | Hannan-Quinn criter. | 0.119147 |
| Durbin-Watson stat | 1.907907 | | |

Dependent Variable: D(LNWHEAT)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/24/12 Time: 10:32
Sample (adjusted): 1552 1697
Included observations: 146 after adjustments
Convergence achieved after 6 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*RESID(-2)^2 + C(7)*@TREND

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| @SQRT(GARCH) | -0.746496 | 0.309706 | -2.410336 | 0.0159 |
| C | 0.186667 | 0.066705 | 2.798375 | 0.0051 |
| AR(2) | -0.217332 | 0.043909 | -4.949605 | 0.0000 |
| | | Variance Equation | | |
| C | 0.057258 | 0.011726 | 4.882911 | 0.0000 |
| RESID(-1)^2 | -0.087587 | 0.028729 | -3.048774 | 0.0023 |
| RESID(-2)^2 | 0.278444 | 0.130705 | 2.130329 | 0.0331 |
| @TREND | -0.000140 | 9.25E-05 | -1.508790 | 0.1314 |

| | | | |
|---|---|---|---|
| R-squared | 0.174512 | Mean dependent var | 0.009510 |
| Adjusted R-squared | 0.162966 | S.D. dependent var | 0.267713 |
| S.E. of regression | 0.244929 | Akaike info criterion | 0.025988 |
| Sum squared resid | 8.578619 | Schwarz criterion | 0.169038 |
| Log likelihood | 5.102885 | Hannan-Quinn criter. | 0.084112 |
| Durbin-Watson stat | 1.885687 | | |

We introduce a time trend in the variance equation to test if there was any improvement in residual variance, but one we took care of the possibly expandable part of prices, the residual variance does not seem to contain any significant trend any more.